# How open charm production and scaling violations probe the rightmost hard BFKL pole exchange

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## Abstract

In 1994 Zakharov and the present authors argued that in color dipole (CD) BFKL approach to DIS excitation of open charm at moderately large $Q^2$ is dominated by hard BFKL exchange. In view of the rapid accumulation of the experimental data on small-$x$ charm structure function of the proton $F_2^c$ from HERA, we subject the issue of dominance of the rightmost hard BFKL pole exchange to further scrutiny. Based on CD BFKL-Regge factorization we report parameter-free predictions for the charm structure function $F_2^c$ and show that the background to the dominant rightmost hard BFKL exchange from subleading hard BFKL and soft-pomeron exchanges is negligible small from real photo-production to DIS at $Q^2 \lesssim$50-100 GeV$^2$. The agreement with the experiment is good and lends strong support for the intercept of the rightmost hard BFKL pole $\Delta_{\text{IP}} = \alpha_{\text{IP}} - 1 = 0.4$ as found in 1994 in the color dipole approach. We comment on the related determination of $\Delta_{\text{IP}}$ from the $x$-dependence of the longitudinal structure function $F_L(x, Q^2)$ and of the scaling violation $\partial F_2 / \partial \log Q^2$ taken at a suitable value of $Q^2$.

# 1 Introduction

In color dipole (CD) approach to small-$x$ DIS excitation of heavy flavor is described in terms of interaction of $q\bar{q}$ color dipoles in the photon with a predominantly small size,

$$\frac{4}{Q^2 + 4m_q^2} \lesssim r^2 \lesssim \frac{1}{m_q^2},$$

(1)

and heavy flavor excitation at large values of the Regge parameter,

$$\frac{1}{x} = \frac{W^2 + Q^2}{4m_c^2 + Q^2} \gg 1,$$

(2)

is an arguably sensitive probe of short distance properties of vacuum exchange in QCD. The first analysis of small-$x$ behavior of open charm structure function (SF) of the proton $F_2^c$ in the color dipole formulation of the Balitsky-Fadin-Kuraev-Lipatov equation [1] has been carried out in 1994 [2, 3, 4] with an intriguing result that for moderately large $Q^2$ it is dominated by hard BFKL exchange. As a matter of fact, the 1994 numerical predictions [4] for $F_2^c$ were in the right ball-park and agree favorably with the recent experimental data from ZEUS Collaboration [5]. Our early observation on hard BFKL dominance in [2, 3] has been based on numerical studies of solutions of our CD BFKL equation [6]; more recently this fundamental feature of CD BFKL approach has been related [7] to nodal properties of eigen-functions of subleading hard BFKL-Regge poles [8]. Here we recall that as noticed by Fadin, Kuraev and Lipatov in 1975 ([9], see also more detailed discussion by Lipatov [10]), incorporation of asymptotic freedom into BFKL equation changes the spectrum of the QCD vacuum exchange to series of isolated BFKL-Regge poles.

The incorporation of the running coupling and imposition of the finite range of propagation $R_c$ of perturbative gluons in our CD BFKL equation provides the interpolation between the BFKL and DGLAP equations. Although it does not necessarily exhaust all infrared cutoffs and resummation of higher order corrections, it emphasizes correctly the principal phenomenon of enhancement of the infrared region by asymptotic freedom and, confirming the Lipatov's analysis [10], provides the splittings of the BFKL spectrum into isolated Regge poles and gives the subleading BFKL eigenfunctions with expected nodal properties, To this end we differ from recent studies [11, 12, 13] of NLO corrections to the original scaling $\alpha_S = const$ and $R_c = \infty$ approximation, in which the emphasis is still on the scaling approximation, the effects of finite $R_c$ have not been incorporated and the full resummation to the running coupling has not been yet completed, for alternative approaches to NLO corrections and infrared effects see [14, 15]. Our approach is closer to that of Ciafaloni, Catani et al., [16] who in their interpolation between the small-$x$ BFKL dynamics and large-$x$ DGLAP dynamics use running coupling in the manner similar to ours. True, the incorporation of asymptotic freedom and going beyond the scaling approximation makes the intercept $\Delta_0$ of the leading BFKL pole sensitive to the infrared regularization, our $\Delta_0 = 0.4$ [8] must be regarded as an educated guess; the principal emphasis is on the nodal properties of subleading solutions and the dependence of an intercept on the number of nodes. To this end it is important that the nodes fall into the perturbative region of small dipoles and are thus controlled by pQCD better than the intercept of the leading pole.

Such a discrete spectrum of QCD vacuum exchange has a far-reaching theoretical and experimental consequences because the contribution of each isolated hard BFKL pole to scattering amplitudes and/or SF's would satisfy very powerful Regge factorization [17]. The resulting CD BFKL-Regge factorized expansion allows one to relate in a parameter-free fashion SF's of different targets, $p, \pi, \gamma, \gamma^*$ [18, 19, 20] and/or contributions of different flavors to the proton SF. In this communication we focus on the latter property of the CD BFKL-Regge factorization and quantify the strength of the subleading hard BFKL and soft-pomeron background to dominant rightmost hard BFKL exchange (to be referred to as LHA for the Leading Hard pole exchange Approximation) improving upon our early somewhat simplified application [7] of the BFKL-Regge factorization to $F_2^c$ and extending the analysis to real photo-production of charm. We find that this background to LHA is small from real photo-production to DIS at $Q^2 \lesssim$ 50-100 GeV$^2$. In view of this fundamental conclusion open charm excitation by real photons and in DIS gives a particularly clean access to the intercept of the rightmost hard BFKL pole for which our 1994 prediction has been

$\Delta_{\mathbf{IP}} = \alpha_{\mathbf{IP}}(0) - 1 \approx 0.4$ [2]. We show how the soft-pomeron background dominant at $Q^2 \lesssim$ 5-10 GeV$^2$ dies out and subleading hard BFKL background builds up for $Q^2 \gtrsim 20$ GeV$^2$. As one could have anticipated, because of the small scale (1) for $c\bar{c}$ color dipoles the soft-pomeron exchange background is negligible small at all $Q^2$ of the practical interest in DIS. Because the CD BFKL-Regge expansion for color dipole-dipole cross section has already been fixed from the related and highly successful phenomenology of light flavor contribution to the proton SF the CD BFKL-Regge factorization predictions for the charm SF of the proton are parameter free. The found nice agreement with the experimental data from ZEUS Collaboration [5] on the charm SF of the proton and open charm photo-production [21, 22] strongly corroborates our 1994 prediction $\Delta_{\mathbf{IP}} = \alpha_{\mathbf{IP}}(0) - 1 \approx 0.4$ for the intercept of the rightmost hard BFKL pole.

Besides charm structure function there are two more observables which are selective to the dipole size: the longitudinal structure function of the proton $F_L$ and the scaling violation slope $\partial F_2 / \partial \log Q^2$ [3]. We present the BFKL-Regge factorization results for these observables. The recent H1 and ZEUS measurements of scaling violation do strongly support $\Delta_{\mathbf{IP}} = 0.4$ [23, 24].

# 2 The selectivity of charm structure function to color dipole radii

In color dipole approach DIS at small $x$ is treated in terms of the interaction of color dipole $\mathbf{r}$ in the photon with the color dipole $\mathbf{r}_p$ in the target proton which is described by the beam ($b$), target ($t$) and flavor independent color dipole-dipole cross section $\sigma(x, \mathbf{r}_b, \mathbf{r}_t)$. The contribution of excitation of open charm to photo-absorption cross section is given by color dipole factorization formula (we suppress the beam, $\gamma^*$, and target, $p$, subscripts in the cross section)

$$\sigma^{c\bar{c}}(x, Q^2) = \int dz d^2\mathbf{r} dz_p d^2\mathbf{r_p} |\Psi_{\gamma^*}^{c\bar{c}}(z, \mathbf{r})|^2 |\Psi_p(z_p, \mathbf{r_p})|^2 \sigma(x, \mathbf{r}, \mathbf{r}') = \int dz d^2\mathbf{r} |\Psi_{\gamma^*}^{c\bar{c}}(z, \mathbf{r})|^2 \sigma(x, \mathbf{r}) .$$
$$(3)$$

where $\sigma(x, \mathbf{r})$ stands for interaction of the beam dipole with the target nucleon. Here $|\Psi_{\gamma^*}^{c\bar{c}}(z, \mathbf{r})|^2$ is a probability to find in the photon the $c\bar{c}$ color dipole with the charmed quark carrying fraction $z$ of the photon's light-cone momentum. The well known result of [25] for the transverse (T) and longitudinal (L) photons is

$$|\Psi_T^{c\bar{c}}(z, r)|^2 = \frac{2\alpha_{em}}{3\pi^2} \left\{ \left[ z^2 + (1-z)^2 \right] \varepsilon^2 K_1(\varepsilon r)^2 + m_c^2 K_0(\varepsilon r)^2 \right\} , \qquad (4)$$

$$|\Psi_L^{c\bar{c}}(z, r)|^2 = \frac{8\alpha_{em}}{3\pi^2} Q^2 z^2 (1-z)^2 K_0(\varepsilon r)^2 , \qquad (5)$$

where $K_{0,1}(y)$ are the modified Bessel functions, $\varepsilon^2 = z(1-z)Q^2 + m_c^2$ and $m_c = 1.5$ GeV is the $c$-quark mass. Hereafter we focus on the charm structure function

$$F_2^c(x_{Bj}, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \int_0^1 dz \int d^2\vec{r} \left[ |\Psi_L^{c\bar{c}}(z, r)|^2 + |\Psi_T^{c\bar{c}}(z, r)|^2 \right] \sigma(x, r)$$
$$= \int \frac{dr^2}{r^2} \frac{\sigma(x, r)}{r^2} W_2(Q^2, m_c^2, r^2) . \qquad (6)$$

We present the results for $F_2^c$ as a function of the conventional Bjorken variable $x_{Bj}$, for the relationship between the Regge parameter $x$ and the Bjorken variable see eq. (13) below.

The Bessel function $K_1(y)$ has the $\sim \frac{1}{y}$ singularity at $y \to 0$ and decreases exponentially at $y \gtrsim 1$, i.e., for color dipole

$$r \gtrsim \frac{1}{\varepsilon}, \tag{7}$$

cf. eq. (1). However, because for small dipoles $\sigma(x,r) \sim r^2$, the dipole size integration in (6) is well convergent at small $r$. A detailed analysis of the weight function $W_2(Q^2, m_c^2, r^2)$ found upon the $z$ integration has been carried out in [3, 4], we only cite the principal results: (i) at moderate $Q^2 \lesssim 4m_c^2$ the weight function has a peak at $r \sim \frac{1}{m_c}$, (ii) at very high $Q^2$ the peak develops a plateau for dipole sizes in the interval (1). One can say that for moderately large $Q^2$ excitation of open charm probes (scans) the dipole cross section at a special dipole size $r_S$ (the scanning radius)

$$r_S \sim \frac{1}{m_c}. \tag{8}$$

The difference from light flavors is that in contrast to the peak for heavy charm the $W_2$ for light flavors always has a broad plateau which extends up to large dipoles $r \sim \frac{1}{m_q}$.

# 3   Scanning radius and nodes of subleading CD BFKL eigen-cross sections and eigen-structure functions

In the Regge region of $\frac{1}{x} \gg 1$ CD cross section $\sigma(x,r)$ satisfies the CD BFKL equation

$$\frac{\partial \sigma(x,r)}{\partial \log \frac{1}{x}} = \mathcal{K} \otimes \sigma(x,r), \tag{9}$$

for the kernel $\mathcal{K}$ of CD approach see [6]. The solutions with Regge behavior

$$\sigma_m(x,r) = \sigma_m(r) \left(\frac{1}{x}\right)^{\Delta_m} \tag{10}$$

satisfy the eigen-value problem

$$\mathcal{K} \otimes \sigma_m = \Delta_m \sigma_m(r) \tag{11}$$

and the CD BFKL-Regge expansion for beam-target symmetric color dipole-dipole cross section reads [2, 19]

$$\sigma(x,r,r_p) = \sum_{m=1} C_m \sigma_m(r) \sigma_m(r_p) \left(\frac{x_0}{x}\right)^{\Delta_m}. \tag{12}$$

The practical calculation of $\sigma(x,r,r_p)$ requires the boundary condition $\sigma(x_0,r,r_p)$ at certain $x_0 \ll 1$. We take for boundary condition at $x = x_0$ the Born approximation,

$$\sigma(x_0,r,r_p) = \sigma_{Born}(r,r_p),$$

i.e. evaluate dipole-dipole scattering via the two-gluon exchange. This leaves the starting point $x_0$ the sole parameter. We follow the choice $x_0 = 0.03$ which met with remarkable phenomenological success [8, 18, 19].

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Here one should not confuse $x$ in the definition of the Regge parameter (2) with the Bjorken variable

$$x_{Bj} = \frac{Q^2}{W^2 + Q^2} = x \cdot \frac{Q^2}{Q^2 + 4m_c^2} . \tag{13}$$

Our choice of normalization of eigen-functions $\sigma_m(r)$ is such that upon calculation of the expectation value over the target proton dipole distribution in (3)

$$\sigma(x, r) = \sum_m \sigma_m(r) \left(\frac{x_0}{x}\right)^{\Delta_m} . \tag{14}$$

The properties of our CD BFKL equation and the choice of physics motivated boundary condition were discussed in detail elsewhere [3, 4, 7, 8, 18], here we only recapitulate features relevant to the considered problem. Incorporation of asymptotic freedom exacerbates well known infrared sensitivity of the BFKL equation and infrared regularization by infrared freezing of the running coupling $\alpha_S(r)$ and modeling of confinement of gluons by the finite propagation radius of perturbative gluons $R_c$ need to be invoked.

The leading eigen-function $\sigma_0(r)$ for ground state. i.e., for the rightmost hard BFKL pole is node free. The subleading eigen-function for excited state $\sigma_m(r)$ has $m$ nodes. We find $\sigma_m(r)$ numerically [8, 18], for the semi-classical analysis see Lipatov [10]. The intercepts (binding energies) follow to a good approximation the law $\Delta_m = \Delta_0/(m + 1)$. For the preferred $R_c = 0.27\,\text{fm}$ as chosen in 1994 in [4, 3] and supported by recent analysis [26] of lattice QCD data we find $\Delta_0 = \Delta_{\mathbb{P}} = 0.4$, the node of $\sigma_1(r)$ is located at $r = r_1 \simeq 0.056\,\text{fm}$, for larger $m$ the rightmost node moves to a somewhat larger $r = r_1 \sim 0.1\,\text{fm}$. The second node of eigen-functions with $m = 2, 3$ is located at $r_2 \sim 3 \cdot 10^{-3}$ fm which corresponds to the momentum transfer scale $Q^2 = \frac{1}{r_2^2} = 5 \cdot 10^3$ GeV$^2$. The third node of $\sigma_3(r)$ is located at $r$ beyond the reach of any feasible DIS experiments. It has been found [8] that the BFKL-Regge expansion (15) truncated at $m = 2$ appears to be very successful in describing of the proton SF's at $Q^2 \lesssim 200$ GeV$^2$. However, at higher $Q^2$ and moderately small $x \sim x_0 = 0.03$ the background of the CD BFKL solutions with smaller intercepts ($\Delta_m < 0.1$) should be taken into account (see below).

The exchange by perturbative gluons is a dominant mechanism for small dipoles $r \lesssim R_c$. In Ref.[4] interaction of large dipoles has been modeled by the non-perturbative, soft mechanism with intercept $\alpha_{\text{soft}}(0) - 1 = \Delta_{\text{soft}} = 0$ i.e. flat vs. $x$ at small $x$. The exchange by two non-perturbative gluons has been behind the specific parameterization of $\sigma_{\text{soft}}(r)$ suggested in [27] and used later on in [7, 8, 18, 19, 20] and here, see also Appendix.

Via equation (6) each hard CD BFKL eigen-cross section plus soft-pomeron CD cross section defines the corresponding eigen-SF $f_m^c(Q^2)$ and we arrive at the CD BFKL-Regge expansion for the charm SF of the proton ($m = \text{soft}, 0, 1, ..$)

$$F_2^c(x_{Bj}, Q^2) = \sum_m f_m^c(Q^2) \left(\frac{x_0}{x}\right)^{\Delta_m} , \tag{15}$$

Now comes the crucial observation that numerically $r_1 \sim \frac{1}{2} r_S$ and the node of hard CD BFKL eigen-cross sections is located within the peak of the weight function $W_2$. Consequently, in the calculation of open charm eigen-SFs $f_m^c(Q^2)$ one scans the eigen-cross section in the vicinity of the node, which leads to a strong suppression of subleading $f_m^c(Q^2)$. This point is illustrated in fig. 1 in which the subleading BFKL-to-rightmost BFKL and soft-pomeron-to-rightmost BFKL ratio of eigen-SFs is shown. For the charm quark mass which is the sole

new parameter we take $m_c = 1.5$ GeV. Because for charm the weight function is peaked at $r \approx r_s$ and, in contrast to that for light flavors, does not extend to larger $r$, the hierarchy and nodal structure of charm eigen-SFs $f_m^c(Q^2)$ differs substantially from that for light flavors discussed in [8]: (i) the node of $f_1^c(Q^2)$ shifts from $Q_1^2 \approx 60\text{GeV}^2$ down to $Q_1^2 \approx 20\text{GeV}^2$, (ii) the first node of $f_2^c(Q^2)$ shifts from $Q_1^2 \approx 30\text{GeV}^2$ down to $Q_1^2 \approx 1\text{GeV}^2$ and $f_2^c(Q^2) \sim 0$ up to $Q \lesssim 20\text{GeV}^2$, (iii) the background SF $f_3^c(Q^2)$ is free of the first node at $Q_1^2 \sim 20\text{GeV}^2$ which is present in eigen-SF for light flavors.

# 4 Predictions from CD BFKL-Regge factorization for open charm structure function and photoproduction

The results shown in fig. 1 form the basis of the CD BFKL-Regge phenomenology of open charm production. Because a probability to find large color dipoles in the photon decreases rapidly with the quark mass, the contribution from soft-pomeron exchange to open charm excitation is very small down to $Q^2 = 0$. In contrast to that for light flavors soft-pomeron exchange was the dominant mechanism at small $Q^2$, see [19]. Large color dipoles are present in the photon and keep contributing to $F_2^c$ even for very large $Q^2$ but relevance of soft-pomeron exchange diminishes gradually with $Q^2$. As we discussed elsewhere [19], for still higher solutions, $m \geq 3$, all intercepts are very small anyway, $\Delta_m \ll \Delta_0$, For this reason, for the purposes of practical phenomenology we can truncate expansion (15) at $m = 3$ lumping in the term $m = 3$ contributions of still higher singularities with $m \geq 3$. The term $m = 3$ which is a combination of higher CD BFKL solutions,

$$\sigma_3(r) = \sigma_{Born}(r) - \sum_{m=0}^{2} \sigma_m(r) \,, \tag{16}$$

is endowed with the effective intercept $\Delta_3 = 0.06$ and is presented in Appendix in its analytical form. Introducing such a term extends the applicability region of the truncated CD BFKL-Regge expansion up to $Q^2 \sim 10^4 - 10^5$ GeV$^2$. Notice that in [19] we accounted for the $m \geq 3$-background in a somewhat different way than that accepted here. However, the difference between two approaches becomes substantial only at $Q^2 \gtrsim 300\,GeV^2$, and do not affect the numerical results at smaller $Q^2$ (mind the Regge suppression factor $(x/x_0)^{\Delta_0-\Delta_m}$).

As fig. 1 shows, the hierarchy of $f_m^c(Q^2)$ is exceptional in that in the very broad of $Q^2$ of the practical interest the contribution from $m = 2$ is negligible small compared to the contribution from $m = 3$. For this reason the term $m = 3$ is numerically important for description of charm structure function at $Q^2 \gtrsim 50$ GeV$^2$. This hierarchy of $f_m^c(Q^2)$ has been overlooked in our early analysis [7] where the truncation of the BFKL-Regge expansion at $m \leq 2$ has been made.

Color dipole cross section is flavor independent and the charm quark mass $m_c = 1.5$ GeV is the sole new parameter in our predictions from the CD BFKL-Regge factorization for open charm SF of the proton presented in fig. 2 as a function of the Bjorken variable $x_{Bj}$ and the results for open charm photoproduction shown in fig. 3. As eq. (13) shows, for small $Q^2$ the starting point $x_0 = 3 \cdot 10^{-2}$ of the BFKL evolution corresponds to progressively smaller $x_{Bj}$ and the CD BFKL-Regge expansion is applicable at

$$x_{Bj} \leq x_0 \cdot \frac{Q^2}{Q^2 + 4m_c^2} \,. \tag{17}$$

and in real photoproduction at

$$\nu \geq \nu_0 = \frac{2m_c^2}{m_p x_0} \sim 150\,\text{GeV}\,. \tag{18}$$

In order to give a crude idea on finite-energy effects at large $x_{Bj}$ and not so large values of the Regge parameter we stretch the theoretical curves a bit to $x \gtrsim x_0$ and/or lower energies $\nu \lesssim \nu_0$ multiplying the BFKL-Regge expansion result (15) by the purely phenomenological factor $(1-x)^5$ motivated by the familiar behavior of the gluon SF of the proton $G(x_{Bj}) \sim (1-x_{Bj})^n$ with the exponent $n \sim 5$.

We comment first on the results on $F_2^c$. The solid curve is a result of the complete CD BFKL-Regge expansion. The long-dashed curve is the pure rightmost hard BFKL pomeron contribution (LHA). The soft (S) pomeron exchange contribution is numerically too small to be shown separately. The sum of the rightmost hard BFKL (LH for the Leading Hard) and soft pomeron exchanges (LHSA) is shown by the dotted curve in the box for $Q^2 = 4$ GeV$^2$ and practically merges with the curve for complete CD BFKL-Regge expansion. This is not unexpected from fig. 1 which shows that for $Q^2 \lesssim 10$ GeV$^2$ there is a strong cancellation between soft and subleading contributions with $m = 1$ and $m = 3$. Consequently, for this dynamical reason in this region of $Q^2 \lesssim 10$ GeV$^2$ we have an effective one-pole picture and LHA gives reasonable description of $F_2^c$. In agreement with the nodal structure of subleading eigen-SFs LHA over-predicts slightly $F_2^c$ at $Q^2 \gtrsim 30$ GeV$^2$, where the negative valued subleading hard BFKL exchanges overtake the soft-pomeron exchange, see fig. 1, and the background from subleading hard BFKL exchanges becomes substantial at $Q^2 \gtrsim 30$ GeV$^2$ and even the dominant component of $F_2^c$ at $Q^2 \gtrsim 200$ GeV$^2$ and $x \gtrsim 10^{-2}$. In this region of $Q^2$ the soft-pomeron exchange is numerically so small the curves for LHSA and LHA merge with each other within the thickness of curves and the LHSA curves are omitted. We predict that open charm SF is dominated entirely by the contribution from the rightmost hard BFKL pole at $Q^2 \lesssim 20$ GeV$^2$, which is due to strong cancellations between the soft-pomeron and subleading hard BFKL exchanges, see fig. 1. The soft-subleading cancellations become less accurate at smaller $x$, but at smaller $x$ the both soft and subleading hard BFKL exchange become rapidly Regge suppressed $\propto x^{\Delta_{\text{IP}}}$, $x^{\frac{1}{2}\Delta_{\text{IP}}}$, respectively.

In fig. 2 we compare our CD BFKL-Regge predictions for small-$x$ charm SF of the proton shown by the solid curve to the recent experimental data from the ZEUS Collaboration [5] and find very good agreement between theory and experiment which lends support to our 1994 evaluation $\Delta_{\text{IP}} = 0.4$ of the intercept of the rightmost hard BFKL pole in the color dipole approach with running strong coupling. The negative valued contribution from subleading hard BFKL exchange is important for bringing the theory to agreement with the experiment at large $Q^2$. Very recently Donnachie & Landshoff have parameterized the same ZEUS data in terms of the two-pole (soft+hard) Regge model [28] and concluded that they are consistent with dominance of the pure hard pole exchange with $\Delta \approx 0.44$. Our dynamical model has more predictive power because it quantifies corrections to single-pole dominance. For instance, it predicts unequivocally that single-pole approximation would break at $Q^2 \gtrsim 50$ GeV$^2$. It also predicts that the background to the rightmost hard BFKL pole with $\Delta_{\text{IP}} = 0.4$ changes from the negligible small value at small $Q^2$ to negative-valued subleading, $m = 3$, hard background BFKL exchange with the intercept $\Delta_3 = 0.06 \approx \frac{1}{7}\Delta_{\text{IP}}$ with a weak for $Q^2$ of the practical interest admixture from subleading, $m = 1$ & $m = 2$, exchanges with larger intercepts $\Delta_1 = 0.22$ and $\Delta_2 = 0.15$. If one would make the effective single-pole fits of the form $F_2^c \propto \left(\frac{1}{x}\right)^{\Delta_{eff}}$, then according to our approach $\Delta_{eff} \sim \Delta_{\text{IP}} = 0.4$

for $Q^2 \lesssim 20$ GeV$^2$ and $\Delta_{eff} \gtrsim \Delta_{\mathbf{IP}}$ and would rise gradually for $Q^2 \gtrsim 20$ GeV$^2$.

In fig. 3 we compare our predictions from CD BFKL-Regge factorization for real photoproduction of open charm with the experimental data from fixed target [21] and HERA collider H1 and ZEUS [22] experiments. The legend of theoretical curves is the same as in fig. 2: the solid curve is a result of the complete BFKL-Regge expansion, the dotted curve is for the Leading Hard + Soft exchange Approximation (LHSA), the long-dashed curve is the pure rightmost hard BFKL pomeron contribution (LHA). The fixed target data are in the region of moderately large Regge parameter when finite-$x$ corrections modeled by the factor $(1 - x)^5$ show up. The agreement between theory and experiment is good and must be regarded as an important confirmation of $\Delta_{\mathbf{IP}} = 0.4$ for the rightmost hard BFKL exchange. For an alternative interpretation of charm photoproduction see [29]).

# 5   Determination of the pomeron intercept $\Delta_{\mathbf{IP}}$ from measurements of $F_L(x, Q^2)$ and $\partial F_2 / \partial \log Q^2$

It has been demonstrated in [3] that the longitudinal structure function $F_L(x, Q^2)$ and the slope of the structure function $\partial F_T / \partial \log Q^2$ emerge as local probes of the dipole cross section at $r^2 \simeq 11./Q^2$ and $r^2 \simeq 2.3/Q^2$, respectively. The subleading CD BFKL cross sections have their rightmost node at $r_1 \sim 0.05 - 0.1$ fm. Therefore, one can zoom at the leading CD BFKL pole contribution and measure the pomeron intercept $\Delta_{\mathbf{IP}}$ from the $x$-dependence of $F_L(x, Q^2)$ at $Q^2 \sim 10 - 30$ GeV$^2$ and of $\partial F_2 / \partial \log Q^2$ at $Q^2 \sim 2 - 10$ GeV$^2$.

In Fig.4 we show the ratio $f_{\mathrm{Lm}} / f_{\mathrm{L0}}$ of subleading to leading longitudinal eigen-SF and soft to leading eigen-SF. From Fig.4 it follows that in the CD BFKL-Regge expansion for $F_L$ (see Appendix) the discussed above cancellation of the soft-subleading contributions is nearly exact at $Q^2 \sim 10 - 30$ GeV$^2$. This results in the leading hard pole dominance in this region (see the box $Q^2 = 20$ GeV$^2$ in Fig.5).

In Fig.6 we presented the ratio $d_m(Q^2)/d_0(Q^2)$ of logarithmic derivatives $d_m(Q^2) = \partial f_m(Q^2)/\partial \log Q^2$ of the all flavor eigen-SF for $m = \mathrm{soft}, 0, 1, 2, 3$ (see [18] for more details). The pattern of cancellations of the soft-subleading contributions is somewhat different in this case and we predict that the leading hard pole dominates the region of several GeV$^2$. The $x_{Bj}$-dependence of the log-derivative $\partial F_2 / \partial \log Q^2$ is shown in Fig.7 for $Q^2 = 0.75, 5$ and $40$ GeV$^2$. The cancellation is exact in the case of $Q^2 \simeq 4$ GeV$^2$. Comparison with preliminary HERA data [23, 24] exhibits good agreement of our calculations with experiment.

# 6   Conclusions

Color dipole approach to the BFKL dynamics predicts uniquely decoupling of subleading hard BFKL exchanges from open charm SF of the proton at $Q^2 \lesssim 20\,\mathrm{GeV}^2$, from $F_L$ at $Q^2 \simeq 20\,\mathrm{GeV}^2$ and from $\partial F_2 / \partial \log Q^2$ at $Q^2 \simeq 4\,\mathrm{GeV}^2$. This decoupling is due to dynamical cancellations between contributions of different subleading hard BFKL poles and leaves us with an effective soft+rightmost hard BFKL two-pole approximation with intercept of the soft pomeron $\Delta_{\mathrm{soft}} = 0$. We predict strong cancellation between the soft-pomeron and subleading hard BFKL contribution to $F_2^c$ in the experimentally interesting region of $Q^2 \lesssim 20$ GeV$^2$, in which $F_2^c$ is dominated entirely by the contribution from the rightmost hard BFKL pole. This makes open charm in DIS at $Q^2 \lesssim 20$ GeV$^2$ a unique handle on the intercept of

the rightmost hard BFKL exchange. Similar hard BFKL pole dominance holds for $F_L(x, Q^2)$ and $\partial F_2 / \partial \log Q^2$. At still higher values of $Q^2$ the soft-pomeron exchange is predicted to die out and negative valued background contribution from subleading hard BFKL exchange with effective intercept $\Delta_3 \approx 0.06$ becomes substantial at not too small $x \sim x_0$. The agreement with the presently available experimental data on open charm in DIS and real photoproduction and the recent data on scaling violation $\partial F_2 / \partial \log Q^2$ is good and confirms the CD BFKL prediction of the intercept $\Delta_{\mathbb{P}} = 0.4$ for the rightmost hard BFKL-Regge pole. The experimental confirmation of our predictions for hierarchy of soft-hard exchanges as function of $Q^2$ would be a strong argument in favor of the CD BFKL approach.

# 7 Appendix

## 7.1 CD BFKL charm eigen-SF

The shape and nodal properties of eigen-functions $\sigma_m(r)$ as a function of $r$ and/or eigen-SFs $f_m^c(Q^2)$ as a function of $Q^2$ is well understood [8, 18, 7]. However, the eigen-cross sections $\sigma_m(r)$ are only available as a numerical solution to the running color dipole BFKL equation. On the other hand, for the practical applications it is convenient to have analytical parameterization for eigen-SFs $f_m^c(Q^2)$, which for the rightmost hard BFKL pole is of the form

$$f_0^c(Q^2) = a_0 \frac{R_0^2 Q^2}{1 + R_0^2 Q^2} \left[ 1 + c_0 \log(1 + r_0^2 Q^2) \right]^{\gamma_0}, \tag{19}$$

where $\gamma_0 = 4/(3\Delta_0)$, while for the subleading hard BFKL poles

$$f_m^c(Q^2) = a_m f_0(Q^2) \frac{1 + K_m^2 Q^2}{1 + R_m^2 Q^2} \prod_{i=1}^{m_{max}} \left( 1 - \frac{z}{z_m^{(i)}} \right), \ m \geq 1, \tag{20}$$

where $m_{max} = \min\{m, 2\}$ and

$$z = \left[ 1 + c_m \log(1 + r_m^2 Q^2) \right]^{\gamma_m} - 1, \ \ \gamma_m = \gamma_0 \delta_m. \tag{21}$$

The parameters tuned to reproduce the numerical results for $f_m^c(Q^2)$ at $Q^2 \lesssim 10^4 \, GeV^2$ are listed in the Table 1.

The soft component of the charm SF as derived from $\sigma_{\text{soft}}(r)$ taken from [27] is parameterized as

$$f_{\text{soft}}^c(Q^2) = \frac{a_{\text{soft}} R_{\text{soft}}^2 Q^2}{1 + R_{\text{soft}}^2 Q^2} \left[ 1 + c_{\text{soft}} \log(1 + r_{\text{soft}}^2 Q^2) \right], \tag{22}$$

with parameters cited in the Table 1.

Table 1. CD BFKL-Regge charm structure functions parameters.

| $m$ | $a_m$ | $c_m$ | $r_m^2$, GeV$^{-2}$ | $R_m^2$, GeV$^{-2}$ | $K_m^2$, GeV$^{-2}$ | $z_m^{(1)}$ | $z_m^{(2)}$ | $\delta_m$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0.02140 | 0.2619 | 0.3239 | 0.2846 | | | | 1. |
| 1 | 0.0782 | 0.03517 | 0.0793 | 0.2958 | 0.2846 | 0.2499 | | 1.9249 |
| 2 | 0.00438 | 0.03625 | 0.0884 | 0.2896 | 0.2846 | 0.0175 | 3.447 | 1.7985 |
| 3 | $-0.26313$ | 2.1431 | $3.7424 \cdot 10^{-2}$ | $8.1639 \cdot 10^{-2}$ | 0.13087 | 158.52 | 559.50 | 0.62563 |
| soft | 0.01105 | 0.3044 | 0.09145 | 0.1303 | | | | |

## 7.2 CD BFKL longitudinal eigen-SF

The CD BFKL expansion for the vacuum component of the all flavor longitudinal SF reads

$$F_L(x_{Bj}, Q^2) = \sum_m f_{Lm}^{uds}(Q^2) \left( \frac{x_0}{x'} \right)^{\Delta_m} + \sum_m f_{Lm}^c(Q^2) \left( \frac{x_0}{x} \right)^{\Delta_m}, \tag{23}$$

where $1/x$ for the charm SF is specified by eq.(2) and the light flavor Regge parameter is

$$\frac{x_0}{x'} = x_0 \frac{Q^2 + W^2}{Q^2 + m_\rho^2} , \tag{24}$$

where $m_\rho$ is the $\rho$-meson mass [18]. The parameterizations for the all flavor longitudinal eigen-SFs $f_{Lm}(Q^2)$ and the longitudinal charm eigen-SFs $f_{Lm}^c(Q^2)$ related to the light flavor eigen-SFs as $f_{Lm}^{uds} = f_{Lm} - f_{Lm}^c$ are presented here. For the all flavor longitudinal eigen-structure functions we have

$$f_{\mathrm{L}0}(Q^2) = a_0 \frac{R_0^2 Q^2}{1 + R_0^2 Q^2} \frac{K_0^2 Q^2}{1 + K_0^2 Q^2} \left[1 + c_0 \log(1 + r_0^2 Q^2)\right]^{\gamma_0} , \tag{25}$$

where $\gamma_0 = \frac{4}{3\Delta_0}$, $R_0^2 = 13.742\,\mathrm{GeV}^{-2}$, $K_0^2 = 0.72578\,\mathrm{GeV}^{-2}$ and

$$f_{\mathrm{L}m}(Q^2) = a_{\mathrm{m}} f_{\mathrm{L}0}(Q^2) \prod_{i=1}^{m} \left(1 - \frac{z}{z_{\mathrm{m}}^{(i)}}\right) , \ m \geq 1 , \tag{26}$$

where

$$z = \left[1 + c_{\mathrm{m}} \log(1 + r_{\mathrm{m}}^2 Q^2)\right]^{\gamma_{\mathrm{m}}} - 1, \ \ \gamma_{\mathrm{m}} = \gamma_0 \delta_{\mathrm{m}} . \tag{27}$$

The parameters adjusted to reproduce the numerical results for $f_{\mathrm{L}\,\mathrm{m}}(Q^2)$ at $Q^2 \lesssim 10^4\,GeV^2$ are listed in the Table 2.

The soft component of the longitudinal structure function is parameterized as

$$f_{\mathrm{L}\,\mathrm{soft}}(Q^2) = a_{\mathrm{soft}} \left(\frac{r_{\mathrm{soft}}^2 Q^2}{1 + r_{\mathrm{soft}}^2 Q^2}\right)^2 \frac{1 + R_{\mathrm{soft}}^2 Q^2}{1 + K_{\mathrm{soft}}^2 Q^2} , \tag{28}$$

with $R_{\mathrm{soft}}^2 = 0.17374\,GeV^{-2}$, $K_{\mathrm{soft}}^2 = 0.61476\,GeV^{-2}$ and $a_{\mathrm{soft}}$, $r_{\mathrm{soft}}^2$ cited in the Table 2.

Table 2. CD BFKL-Regge longitudinal structure functions parameters.

| m | $a_{\mathrm{m}}$ | $c_{\mathrm{m}}$ | $r_{\mathrm{m}}^2$ GeV$^{-2}$ | $z_{\mathrm{m}}^{(1)}$ | $z_{\mathrm{m}}^{(2)}$ | $z_{\mathrm{m}}^{(3)}$ | $\delta_{\mathrm{m}}$ | $\Delta_{\mathrm{m}}$ |
|---|---|---|---|---|---|---|---|---|
| 0 | $9.756 \cdot 10^{-3}$ | 0.24835 | 0.5193 | | | | 1. | 0.402 |
| 1 | 0.34897 | $3.5370 \cdot 10^{-2}$ | 9.6065 | 4.5613 | | | 2.5472 | 0.220 |
| 2 | 0.27132 | $1.8934 \cdot 10^{-2}$ | 5.8656 | 1.9627 | 12.172 | | 3.7111 | 0.148 |
| 3 | 2.38323 | $2.3467 \cdot 10^{-3}$ | 5.1690 | $7.2783 \cdot 10^{-2}$ | 0.20309 | 0.33768 | 2.6115 | 0.06 |
| soft | 0.03181 | | 7.8172 | | | | | 0. |

## 7.3  CD BFKL longitudinal charm eigen-SF

For the longitudinal charm eigen-structure functions the parameterization reads

$$f_{\mathrm{L}0}^c(Q^2) = a_0 \frac{R_0^2 Q^2}{1 + R_0^2 Q^2} \frac{K_0^2 Q^2}{1 + K_0^2 Q^2} \left[1 + c_0 \log(1 + r_0^2 Q^2)\right]^{\gamma_0} , \tag{29}$$

where $\gamma_0 = \frac{4}{3\Delta_0}$ and

$$f_{\mathrm{L}m}^c(Q^2) = a_{\mathrm{m}} f_{\mathrm{L}0}^c(Q^2) \frac{1+R_m^2 Q^2}{1+K_m^2 Q^2} \prod_{i=1}^{m_{max}} \left(1 - \frac{z}{z_{\mathrm{m}}^{(i)}}\right) , \ m \geq 1 , \tag{30}$$

where $m_{max} = \min\{2, m\}$,

$$z = \left[1 + c_{\mathrm{m}} \log(1 + r_{\mathrm{m}}^2 Q^2)\right]^{\gamma_{\mathrm{m}}} - 1, \ \ \gamma_{\mathrm{m}} = \gamma_0 \delta_{\mathrm{m}} . \tag{31}$$

The parameters adjusted to reproduce the numerical results for $f_{\mathrm{L}\,m}^c(Q^2)$ at $Q^2 \lesssim 10^4 \ \mathrm{GeV}^2$ are listed in the Table 3.

The soft component of the longitudinal charm structure function is parameterized as

$$f_{\mathrm{L}\,\mathrm{soft}}^c(Q^2) = a_{\mathrm{soft}} \left(\frac{r_{\mathrm{soft}}^2 Q^2}{1+r_{\mathrm{soft}}^2 Q^2}\right)^2 , \tag{32}$$

with parameters cited in the Table 3.

Table 3. CD BFKL-Regge longitudinal charm structure functions parameters.

| m | $a_{\mathrm{m}}$ | $c_{\mathrm{m}}$ | $r_{\mathrm{m}}^2$ $\mathrm{GeV}^{-2}$ | $R_{\mathrm{m}}^2$ $\mathrm{GeV}^{-2}$ | $K_{\mathrm{m}}^2$ $\mathrm{GeV}^{-2}$ | $z_{\mathrm{m}}^{(1)}$ | $z_{\mathrm{m}}^{(2)}$ | $\delta_{\mathrm{m}}$ |
|---|---|---|---|---|---|---|---|---|
| 0 | $7.8617 \cdot 10^{-3}$ | 0.17919 | 0.41493 | 0.33040 | 0.05012 | | | 1. |
| 1 | 0.14496 | $7.0144 \cdot 10^{-2}$ | 0.12531 | 0. | 0. | 0.90916 | | 1.5407 |
| 2 | $4.7714 \cdot 10^{-2}$ | $2.5041 \cdot 10^{-2}$ | 0.10782 | 0. | 0. | 0.21016 | 5.7923 | 3.1029 |
| 3 | $-0.22432$ | 1.1516 | 0.027011 | 0.20426 | 0.089174 | 40.533 | 213.34 | 0.65636 |
| soft | $3.4956 \cdot 10^{-3}$ | | 0.10374 | | | | | |

## 7.4 CD BFKL all flavor eigen-SF

Here we represent the results of numerical solutions for the all flavor eigen-SF which is the sum of longitudinal and transverse eigen-SF

$$f_m(Q^2) = f_{\mathrm{L}m}(Q^2) + f_{\mathrm{T}m}(Q^2)$$

in an analytical form

$$f_0(Q^2) = a_0 \frac{R_0^2 Q^2}{1+R_0^2 Q^2} \left[1 + c_0 \log(1 + r_0^2 Q^2)\right]^{\gamma_0} , \tag{33}$$

$$f_m(Q^2) = a_m f_0(Q^2) \frac{1+R_0^2 Q^2}{1+R_m^2 Q^2} \prod_{i=1}^{m} \left(1 - \frac{z}{z_m^{(i)}}\right) , \ m \geq 1 , \tag{34}$$

where $\gamma_0 = \frac{4}{3\Delta_0}$ and

$$z = \left[1 + c_m \log(1 + r_m^2 Q^2)\right]^{\gamma_m} - 1, \ \ \gamma_m = \gamma_0 \delta_m . \tag{35}$$

The parameters tuned to reproduce the numerical results for $f_m(Q^2)$ at $Q^2 \lesssim 10^4 \, GeV^2$ are listed in the Table 4.

Table 4. CD BFKL-Regge structure functions parameters.

| $m$ | $a_m$ | $c_m$ | $r_m^2$, GeV$^{-2}$ | $R_m^2$, GeV$^{-2}$ | $z_m^{(1)}$ | $z_m^{(2)}$ | $z_m^{(3)}$ | $\delta_m$ |
|------|--------|---------|---------|---------|---------|---------|---------|--------|
| 0 | 0.0232 | 0.3261 | 1.1204 | 2.6018 | | | | 1. |
| 1 | 0.2788 | 0.1113 | 0.8755 | 3.4648 | 2.4773 | | | 1.0915 |
| 2 | 0.1953 | 0.0833 | 1.5682 | 3.4824 | 1.7706 | 12.991 | | 1.2450 |
| 3 | 1.4000 | 0.04119 | 3.9567 | 2.7706 | 0.23585 | 0.72853 | 1.13044 | 0.5007 |
| soft | 0.1077 | 0.0673 | 7.0332 | 6.6447 | | | | |

The soft component of the proton structure function is parameterized as follows

$$f_{\text{soft}}(Q^2) = \frac{a_{\text{soft}} R_{\text{soft}}^2 Q^2}{1 + R_{\text{soft}}^2 Q^2} \left[ 1 + c_{\text{soft}} \log(1 + r_{\text{soft}}^2 Q^2) \right] , \tag{36}$$

with parameters cited in the Table 4.

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**Figure captions**

**Fig.1** The subleading hard-to-rightmost hard and soft-pomeron-to-rightmost hard ratio of eigen-structure functions $f_m^c(Q^2)/f_0^c(Q^2)$ as a function $Q^2$.

**Fig.2** Prediction from CD BFKL-Regge factorization for the charm structure function of the proton $F_2^c(x, Q^2)$ as a function of the Bjorken variable $x_{Bj}$ in comparison with the experimental data from ZEUS Collaboration [5]. The solid curve is a result of the complete CD BFKL-Regge expansion, the contribution of the rightmost hard BFKL pole (LHA) with $\Delta_{\mathbf{IP}} = 0.4$ is shown by long-dashed line. The dotted curve in the box for $Q^2 = 4$ GeV$^2$ shows a sum of the rightmost hard BFKL plus soft-pomeron exchanges (LHSA). The upper long-dashed curve in each box for $Q^2$ from 1.8 GeV$^2$ up to 30 GeV$^2$ corresponds to LHA at $m_c = 1.3$ GeV. At higher $Q^2$ the effect of variation of $m_c$ from 1.5 to 1.3 GeV is negligible small.

**Fig.3** Predictions from CD BFKL-Regge factorization for open charm photoproduction cross section $\sigma(\gamma p \to c\bar{c}X)$. The solid curve is a result of the complete CD BFKL-Regge expansion, the contribution of the rightmost hard BFKL pole (LHA) with $\Delta_{\mathbf{IP}} = 0.4$ is shown by long-dashed line. The dotted curve shows a sum of the rightmost hard BFKL plus soft-pomeron exchanges (LHSA). The upper dotted curve corresponds to the LHSA with $m_c = 1.3$ GeV. The data points are from fixed target [21] and H1&ZEUS HERA [22] experiments.

**Fig.4** The subleading hard-to-rightmost hard and soft-pomeron-to-rightmost hard ratio of longitudinal eigen-structure functions $f_{\mathrm{Lm}}(Q^2)/f_{\mathrm{L0}}(Q^2)$ as a function of $Q^2$.

**Fig.5** Prediction from CD BFKL-Regge factorization for the longitudinal structure function of the proton $F_L(x_{Bj}, Q^2)$ as a function of the Bjorken variable $x_{Bj}$. The solid curve is a result of the complete CD BFKL-Regge expansion, the contribution of the rightmost hard BFKL pole (LHA) with $\Delta_{\mathbf{IP}} = 0.4$ is shown by long-dashed line. The dashed curve shows the soft-pomeron contribution.

**Fig.6** The subleading hard-to-rightmost hard and soft-pomeron-to-rightmost hard ratio of logarithmic derivatives $d_{\mathrm{m}}(Q^2)/d_0(Q^2)$ of the all flavor eigen-SF $d_m(Q^2) = \partial f_m/\partial \log Q^2$ as a function of $Q^2$.

**Fig.7** Prediction from CD BFKL-Regge factorization for the log-derivative of the proton structure function $\partial F_2/\partial \log Q^2$ as a function of the Bjorken variable $x_{Bj}$. The solid curve is a result of the complete CD BFKL-Regge expansion, the contribution of the rightmost hard BFKL pole (LHA) with $\Delta_{\mathbf{IP}} = 0.4$ is shown by long-dashed line. The dashed curve shows the soft-pomeron contribution. Preliminary data by H1 [23] and ZEUS [24] are shown by filled and open triangles, respectively.